Overturn study in the moon thermal evolution

Andrea Combette, Louis Poitevin

ENS Lyon

January 10, 2025

Overview

- Moon formation : giant impact between Earth and a Mars-sized body
- fully molten moon
- cooling and solidification in two steps
  - *radiative cooling with olivine-pyroxene cumulates*
  - *formation of anorthite crust (diffusion cooling)*

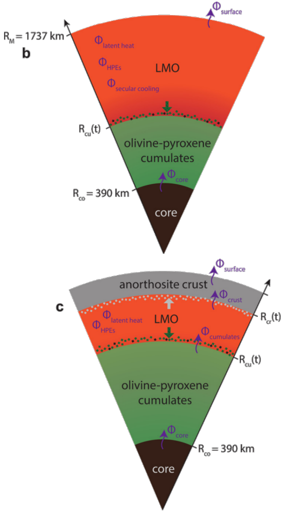


Figure – Moon formation

Radiative cooling

- Consider only a mix of anorthite and olivine-pyroxe

$$T_{\text{liq}} = T_{\text{OL}} - mC(t), \quad T_{\text{LMO}} = T_{\text{liq}}$$

- The conservation of anorthite yields :

$$(R_M^2 - R_{\text{co}}^2)C_0 = (R_M^3 - R_{\text{cu}}^3)C(t)$$

- We end up with the following :

$$T_{\text{liq}}(t) = T_{\text{OL}} - mC_0 \frac{R_M^3 - R_{\text{co}}^3}{R_M^3 - R_{\text{cu}}(t)^3}$$

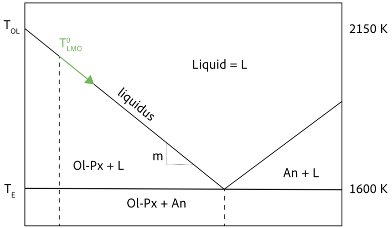


Figure – Phase diagram Olivine-pyrox / Anorthite

Radiative cooling

- Assuming that $T_{cu(r)} = T_{liq}(R_{cu}(t) = r)$ due to the short time scale of the first stage (neglect diffusion in the cumulate)
  - $10^2 \sim 10^3 yr$
  - *instable temperature profile*
- When $C(t) = C_E$ the anorthite crust is formed and slow down the cooling
- The cristiallisation of the olivine is then slowed down (we can consider a constant width cumulates layer)
- However, this is instable density profile

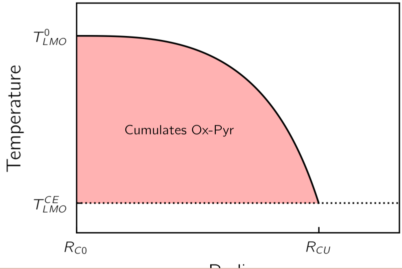


Figure – Instable temperature profile leading to overturn and increasing in flux

Modeling of the system

- We are interested in the dynamic of this cumulate layer at the beginning of the second stage
- The flux will increase and the temperature profile will be stable
- Convection in the 2d slab **Boussinesq** approximation with free slip boundary layer

$$\begin{cases} \vec{\nabla} \cdot \vec{u} = 0 \\ \rho_0 \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u} = \vec{\nabla} P + \eta \nabla^2 \vec{u} + \rho \vec{g} \\ \frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa \nabla^2 T \\ \rho = \rho_0 (1 - \alpha (T - T_0)) \end{cases}$$

We expect to have thermal conduction driven Rayleigh-Bénard convection. This will leads to the following rescaling, filtering out the short time-scales:

$$\hat{x}, \hat{y} = \frac{\hat{x}}{d}, \frac{\hat{y}}{d} \quad \hat{z} = \frac{z}{d} + \frac{1}{2} \quad \hat{\theta} = \frac{\theta}{\Delta T_0} \quad \hat{t} = \frac{t\kappa}{d^2} \quad \hat{p} = \frac{pd^2}{\kappa \eta}$$

Modeling of the system

- We end up with the following dimensionless equations dropping the hats and considering infinite Prandtl number :

$$\begin{cases} \vec{\nabla} \cdot \vec{u} = 0 \\ \frac{1}{\text{Pr}} \frac{Du}{Dt} = -\vec{\nabla} p + \nabla^2 \vec{u} + Ra\theta \vec{e}_z = 0 \\ \frac{D\theta}{Dt} = \nabla^2 \theta \end{cases}$$

  With $\text{Ra} = \frac{\rho_0 g \alpha \Delta T d^3}{\kappa \eta}$ and $\text{Pr} = \frac{\eta}{\kappa \rho_0}$. The infinite Prandtl approximation leads to a momentum dissipation greater than the thermal dissation, the velocity field will then react immediately to a change of temperatures.

- The following boundary conditions are considered :
  - $\vec{u} \cdot \vec{e}_z = 0$ *on* $z = 0, 1$ *(impermeability)*
  - $\frac{\partial \theta}{\partial z} = 0$ *on* $z = 0$ *(negligeable flux induced by the core)*
  - $\tau^x = 0$ *on* $z = 0, 1$ *(Free-slip)*
  - $\theta = 0$ *on* $z = 1$ *(Constant Temperature of the LMO)*

Motivation

- The goal of this project is to study the evolution of the cumulate layer at the beginning of the second stage of the moon cooling depending on the **temperature profiles**and on the range of **Ra**
- Study of the depth of the temperature profile and its impact on the thermal flux
- The moon cumulates Ra should be in the range $10^5 \sim 10^6$
  - *impact on the dynamics*
  - *this overturn flux is usually taken as :* $\Phi_{OV} \sim e^{-\frac{t}{\tau_{OV}}}$
  - *study this scaling law*

Numerical Setup

- Semi spectral method for convection-diffusion equations
- Fourrier basis for the x basis (Periodic Boundary conditions)
- Chebyshev basis for the z basis (Allowing non-periodic boundary conditions)
- CGL nodes for the Chebyshev basis and equally spaced nodes for the Fourier basis
- Time integration using a second order Runge kutta scheme with adaptive time stepping

Initial Thermal conditions

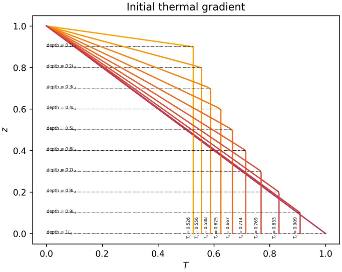


Figure – Initial temperature profile

- we ensure that the integral over all the depth is constant for all profile ensuring a constant amount of energy among all the profiles
- $$T(z > e) = T_e + \frac{1 - z}{e(2 - e)}$$

Average temperature - Nusselt & Reynolds

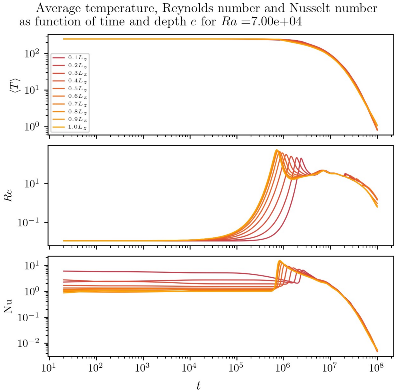


Figure – Average temperature profile Nusselt and Reynolds number for Ra $= 7.10^4$

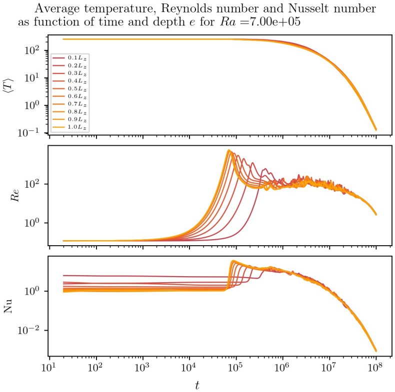
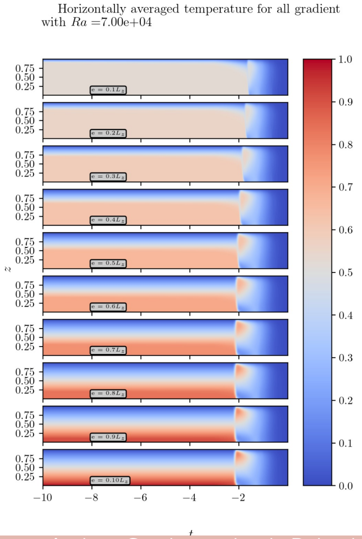
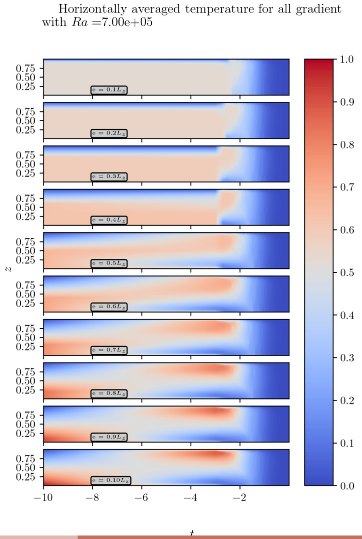


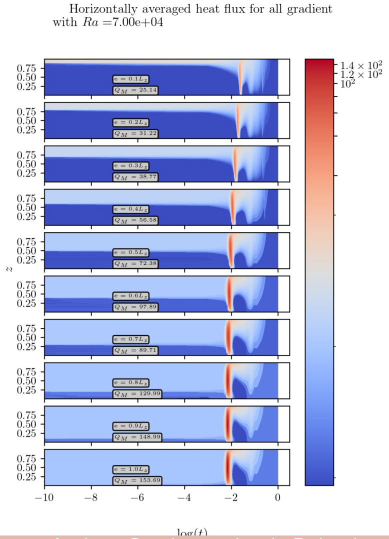
Figure – Average temperature profile Nusselt and Reynolds number for Ra $= 7.10^5$

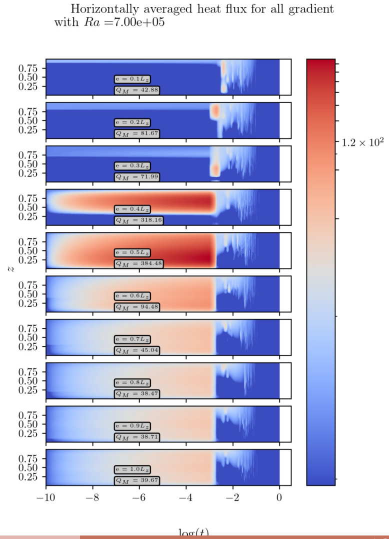
Average temperature - Nusselt & Reynolds





Overturn-Flux profile





Overturn-Flux profile

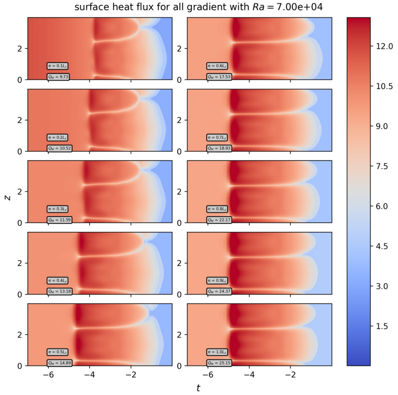


Figure – Surface heat flux for Ra = 7.10^4

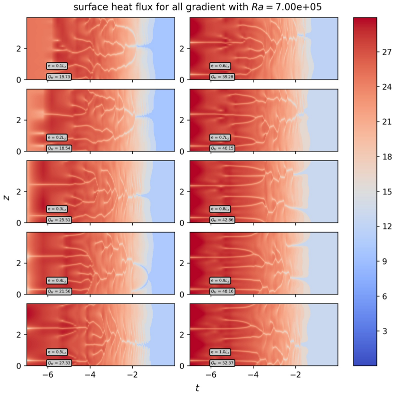
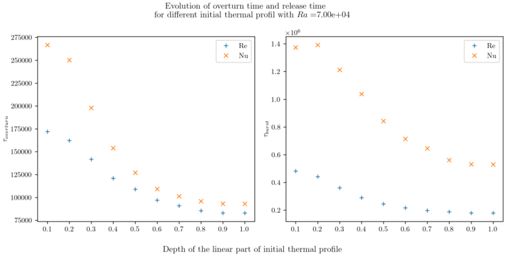
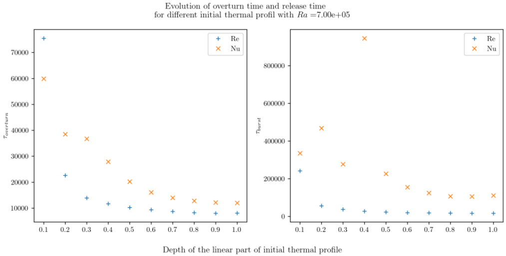


Figure – Surface heat flux for Ra = 7.10^5

Overturn and burst duration





Overturn and burst duration

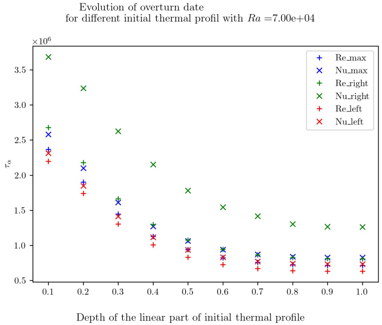


Figure – Overturn time for Ra = 7.10^4

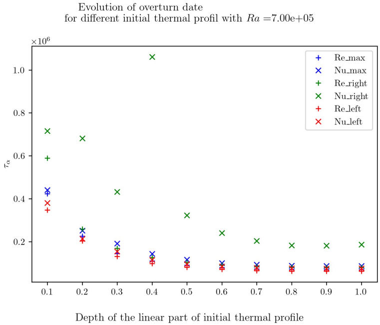


Figure – Overturn time for Ra = 7.10^5

Maximum Heat flux density in cumulates

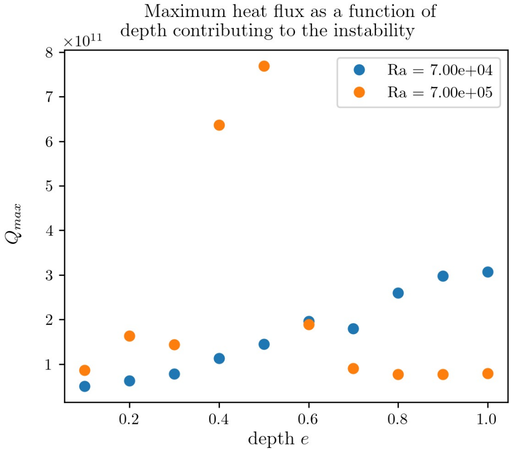
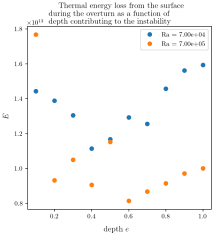
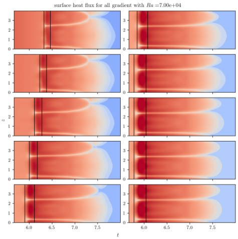
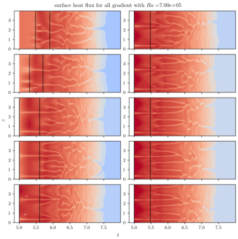


Figure – Maximum heat flux contribution for Ra $= 7.10^4$ and 7.10^5

Loss of energy







Overturn and Burst duration

- Since the the overturn time and duration should be dependant of the convective time, the overturn time is dependant of the Rayleigh.
- More precisely, we expect the following scaling law :

$$\tau_{\mathrm{OV}} \sim \tau_{\mathrm{conv}} = \mathrm{Ra}^{-1}\tau_{\mathrm{diffusion}}$$

- For this we can study the total flux : $\Phi_{\mathrm{tot}}(z=0) = V\theta - \partial_z\theta$

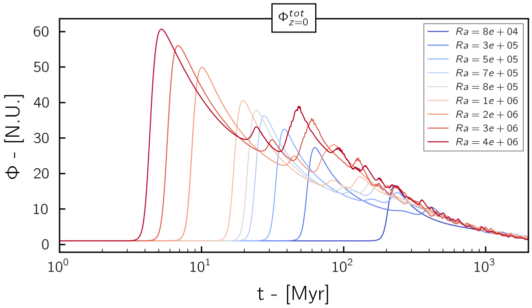


Figure – Overturn Dynamics for different Ra.

Overturn and Burst duration

The overturn time and the duration of the overturn dependancy are plotted bellows :

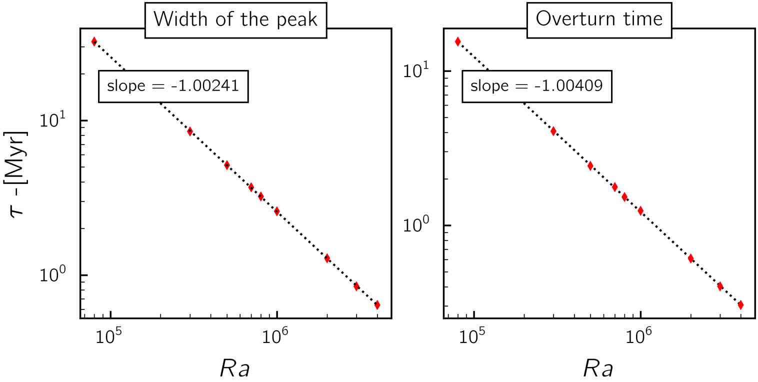


Figure – The overturn characteristic times are only convective scaling like predicted as Ra^{-1}, a diffusive behavior would have led to a scaling like Ra^1

Overturn and Burst duration

- One could suggest to rescale the time using the Rayleigh number to have comparable dynamics

$$\hat{t} = \mathrm{Ra} \cdot t$$

- This unearthes very close dynamics for the different Rayleighs which seems to play a role only on the intensity of the convection events.

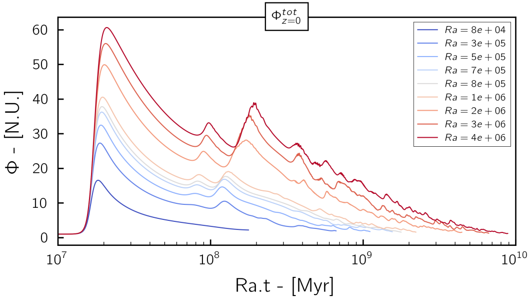


Figure – Rescaled overturn dynamics for different Ra.

Nusselt and Total Heat flux

- Next, we have to study the impact of the Ra on the intensity of the dynamics
- for large Rayleigh with overturn free dynamics one could unearth that the adimensional flux Nu verifies : $Nu \sim \mathrm{Ra}^{\frac{1}{3}}$
- The nusselt number can be expressed as the ratio of the total flux and the condictive flux
$$Nu = \frac{\Phi_{\mathrm{tot}}}{\Phi_{\mathrm{cond}}} = \frac{V\theta - \partial_z \Theta}{\partial_z \Theta}$$
- in our study we have at $z = 0$, $v = 0$, we deduce easily that $Nu = 1$, and that the flux is only diffusive at the boundary. The study of the total flux will then be much more intesting.

Nusselt and Total Heat flux

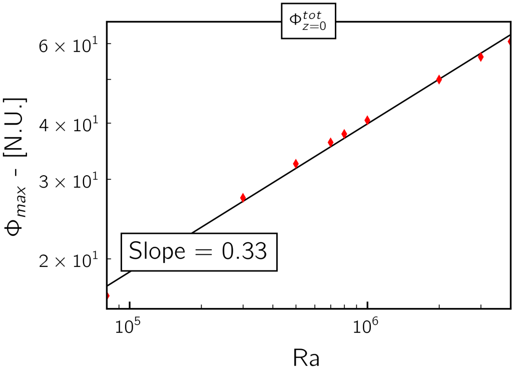
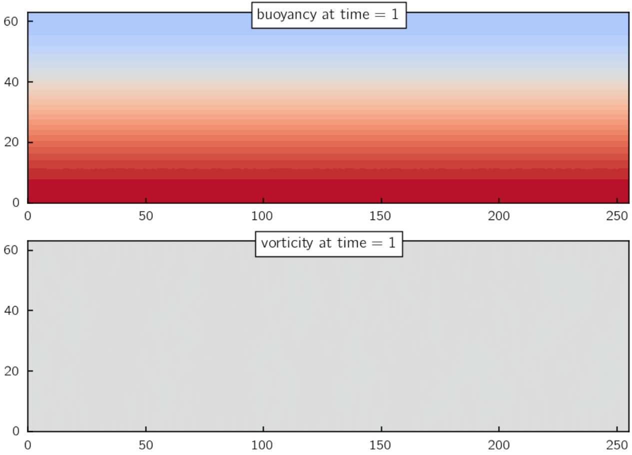


Figure – Φ^{max} scaling over the Raynold. We recover the previous law $Nu = \text{Ra}^{\frac{1}{3}}$

Modal analysis



Modal analysis

- One could also study the convection events we saw before with multiple overturns occuring.
- For that we have to recall the first instable modes has a wavevector of value :
  - $k = 2.23$ *for the free-free Boundary conditions*
  - $k = 3.12$ *for the rigid-rigid Boundary conditions (even modes)*

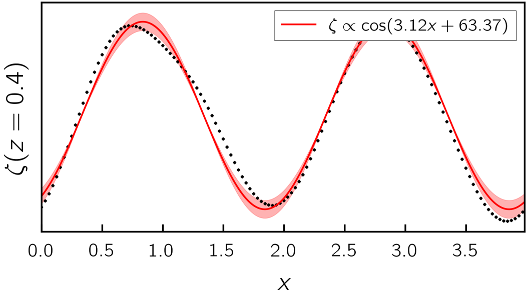
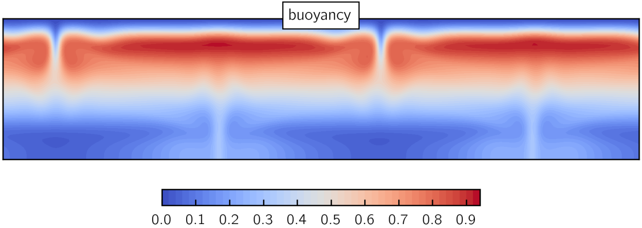
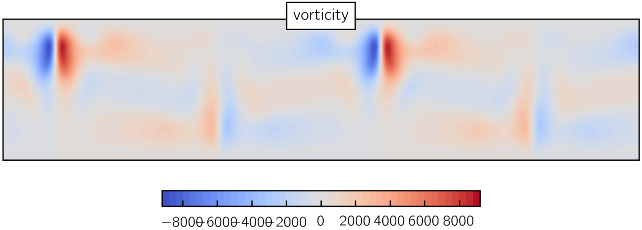


Figure – Vorticity profile before the first overturn, it seems that the BC are rigid (Pr infinite and $\theta(z = 0) = 0$

Modal analysis

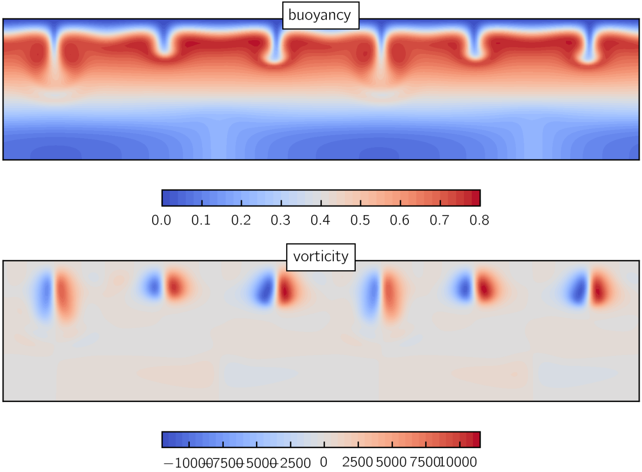
Fields after the first overturn





Modal analysis

Fields just before the second overturn



Modal analysis

- All the natural frequencies of the system are then multiples of the first instable mode, implying a cascade of convective cell fusion or division at different scales (This explains the charcteristic cooling of the system that is not exponential).

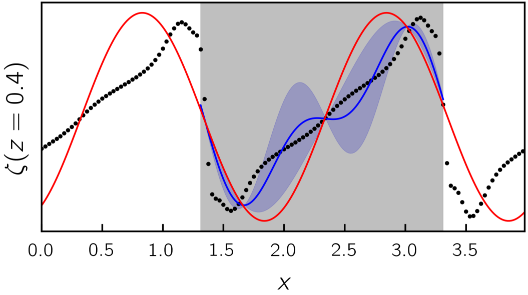
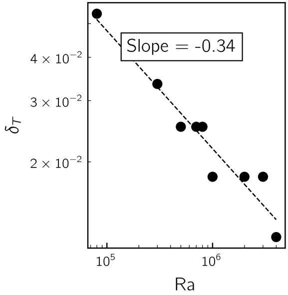


Figure – Vorticity profile after the first overturn, the BC are still rigid

Cooling rate

- The cooling is localized in a boundary layer at the top of the Box where the buoyancy is equilibrated by the diffusion processes.

$$\delta_T = \frac{L}{\mathrm{Ra}^{\frac{1}{3}}}$$



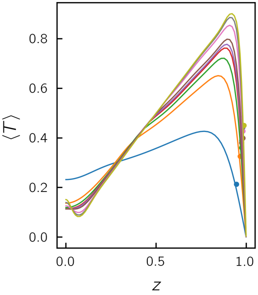


Figure – Temperature profile and boundary layer

□

Cooling rate

- Still have to determine the decaying scaling