Master Thesis report M1 ICFP, ENS Paris

Prediction of the density fluctuation level using machine learning applied to the Short Pulse Reflectometry data

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Plasma Turbulence

- anomalous transport in tokamak
- limit our study to Trapped Electron Mode (TEM)
- resonant interaction between plasma drift waves and trapped electrons
- corrugations in the plasma electronic density profile

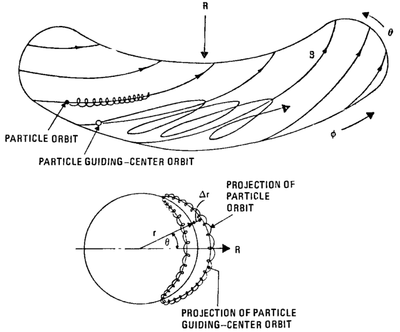


Figure – Trapped particle motion in the Tokamak exhibits a banana shape. If a wave resonates with the particle at a lower frequency than the particle's transit time, the particle can exchange energy

Short Pulse Reflectometry

- probing the plasma with a very short microwave pulse (some nanoseconds), which reflects off the cutoff region of the plasma
- determined by the electron density profile
- two mode of wave propagation : $\mathcal{O}$-mode and $\mathcal{X}$-mode
- limit our study to $\mathcal{O}$-mode

- two ways of solving the wave equation
  - *Wentzel–Kramers–Brillouin (WKB) approximation*
  - *full wave numerical simulation (CUWA)*[a]
- 2 ways will be studied
- inferring the turblences amplitude from the reflected pulse characteristics in specific regime.

[a] P. Aleynikov and N. B. Marushchenko, "3d full-wave computation of rf modes in magnetised plasmas", Computer Physics Communications **241**, 40–47 (2019).

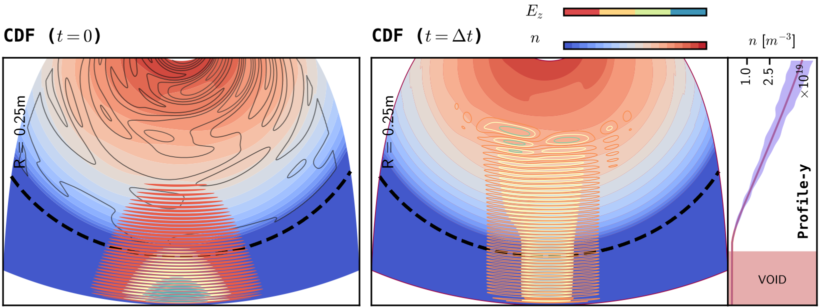


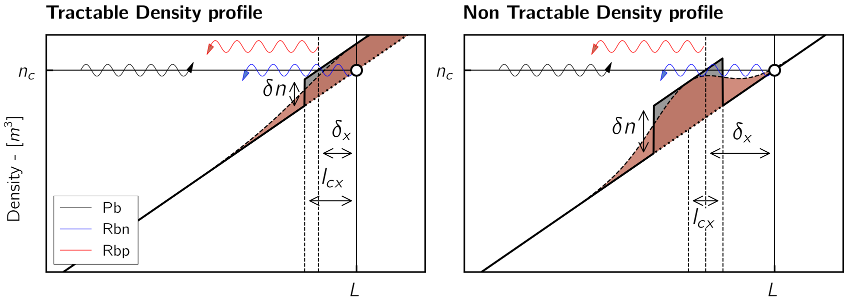
Figure – SPR setup . The probing wave is sent to the plasma, and the reflected wave from the cut-off is measured. The delay between the two waves provides a measure of the plasma density profile.

Analytic Model

- 1d density profile n(x), WKB approximation, to link the delay of the probing wave to the density profile.
- The delay of the probing wave is given by the following formula:

$$\tau_d = 2 \int_0^L \frac{dx}{v_g},$$

where v_g is the group velocity of the wave.



- condidering a singe step like turbulences we got :

$$\tau_d = 2\int_0^L \frac{dx}{v_g} = \frac{4L}{c} - \frac{2L}{c}\sqrt{\frac{L}{l_{cx}}}\frac{\delta n}{n_c}.$$

- statistical approach δn as a random variable
- standard deviation of the delay depending on the standard deviation of perturbations. This yields:

$$\sigma_{\tau_d} \approx \frac{2L}{c}\sqrt{\frac{L}{l_{cx}}}\frac{\sigma_{\delta n}}{n_c}.$$

- testing by comparing the analytical expression with the numerical integration of the wave equation for numerous Gaussian perturbations for several parameters

$$\delta n(k_x) \propto \delta n_0 \exp\left(-\frac{(k_x l_{cx})^2}{8} + i\Phi(k_x)\right)$$

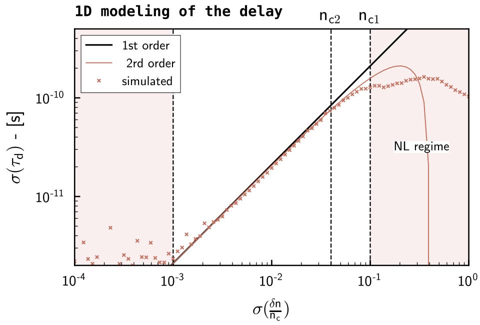


Figure – 1-dimensional predicted amplitude of the analytical 1st and 2nd order step-driven model, compared to the simulated 1-dimensional delay. A constant δn_0 has been used for the density profile.

- **Results**
  - *good agreement in the linear regime*
  - $$\delta n \geq n_c \frac{l_{cx}}{L} = n_{c1}$$ $$\frac{\delta n}{n_c} \gg \frac{c}{w\sqrt{l_{cx} L \ln \frac{L}{l_{cx}}}} = n_{c2}$$
  - *characteristic saturation of the standard deviation in non-linear regime*
- **Discussion**
  - *the non-linear regime is significant in experimental conditions*
  - *build a model tackling the non-linear regime*
  - *find relevant parameters for the model*

Reliable parameters

- for the previous model we just used the standard deviation
- for this model we need to limit to delay distribution
- quantiles and moments of the delay distribution as parameter for our model

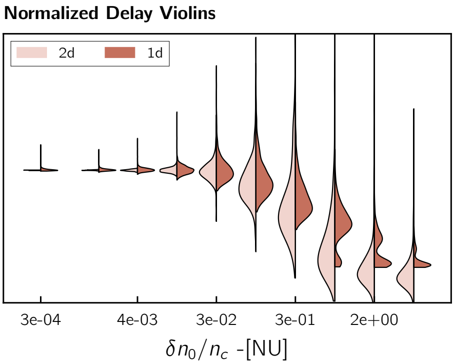


Figure – Violin plots of the delay distribution over δn_0

Model

- machine learning model
- trained to predict δn_0 and τ_0
- L, l_{cx}, $\langle \tau_d \rangle$, σ_{τ_d} and quantiles as input
- stacked Multi Output Regressor

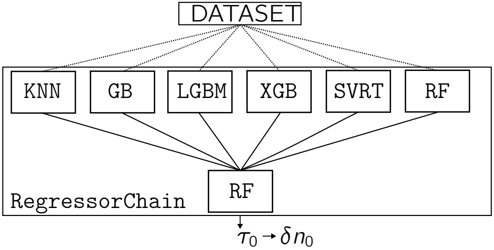


Figure – Stacked Regressor structure with a global regressor chain. Models combination is key to the performance of our predictions, especially in extreme cases with high or low turbulence amplitudes.

Results

- very good results $R2$ of 0.94 for the model on the 1d datasets
- τ_0 is better predicted than δn_0
- poor results on 2d datasets, even while shifted to the mean of the 1d datasets

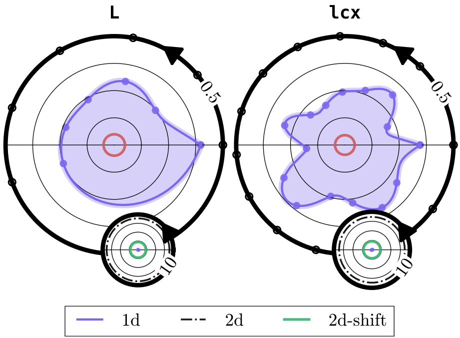


Figure – Plot of the amplitude residuals of the model for the 1D testing set and the 2d sets

- this model does not take into account multiple scattering
- need to build a 2d model to tackle incidence angle and curvature effect
- no pulse shape information in this 1d model
- give an overview of the possible efficiency of the model with just the delay distribution

2d model

- the 2d model is based on the 1d model
- the dataset will not be built using the WKB approximation and the numerical integration
- based on the full wave numerical simulation (**CUWA**)

Metrics

- carry the most information about the pulse shape
- broadening of the pulse and decrease of the amplitude
- dispersion and scattering effect

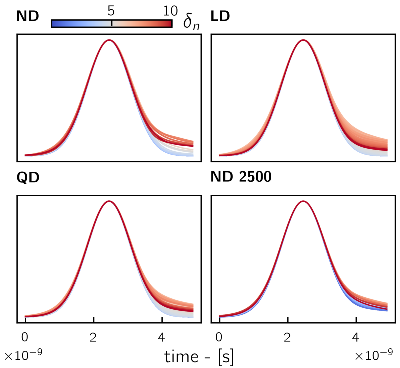


Figure – Normalized mean of centered reflected pulse signal for several density profiles.

- skewness of the delay distribution
- asymmetry of the pulse shape
- governed by the second critical density
- broadening of the pulse was mainly due to the increasing number of spikes, i.e the multiple scatterings

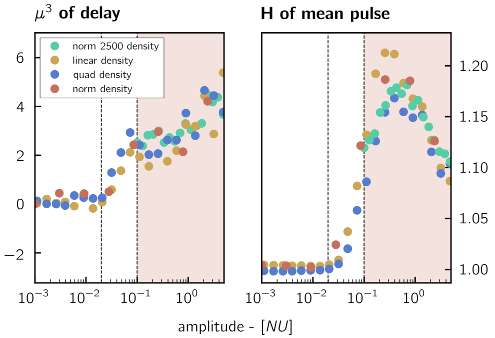


Figure – Here we computed the skewness μ^3 and the asymmetry H of the mean pulse.

Datasets Building

- quantiles' distribution of the delay and moments
- mean pulse amplitude, mean asymmetry, skewness of the mean pulse, standard deviation of the pulse width
- generating $\delta n(\mathbf{k})$ field and IFFT
- gaussian and power spectrum of the turbulences

- point selection (random and grid)
- **CUWA** simulations on LEONARDO
- storage and processing of the datasets in SQL database

Gaussian Spectrum

$$\delta n(\mathbf{k}) \propto \delta n_0 \exp\left(-\frac{(k_x l_{cx})^2 + (k_y l_{cy})^2}{8} + i\Phi(\mathbf{k})\right)$$

- useful to first create a dataset with controlled correlation length
- not the real spectrum of the turbulence

Power Spectrum

- non separability of k_x k_y.

$$\langle \delta n^2 \rangle = \frac{1}{1 + \left|\frac{k_x}{W_x}\right|^{\gamma} + \left|\frac{k_y - k_y^*}{W_y}\right|^{\beta}}$$

- correlation length are not directly determined
- we tried to find the analytical expression of l_{cx}, l_{cy} with Wiener theorem

$$l_{cx} \propto \left(\frac{1}{W_x C^{1/\gamma}}\right) , \; l_{cy} \propto \left(\frac{1}{W_y Z^{1/\beta}}\right)$$

- numerical linear fit and integration
- compared with calculated cross correlation function in the normal and Wiener way

$$r_{xx}(\tau) = \frac{\sum_s \left(\delta\tilde{n}_s(x+\tau, y)\right)\left(\delta\tilde{n}_s(x, y)\right)}{\sum_s \left(\delta\tilde{n}_s(x, y)\right)^2}$$

$$r_{xx}(\tau) = \int_{-\infty}^{\infty} < \delta n(k_x, k_y)^2 > e^{2\pi k_x \tau} \, dk_x$$

Data Scanning

- gridded and random points for gaussian to begin the learning process
- was the training set coverage satisfying?
- introducing high value of R and l_{cy} to mimic the 1d case

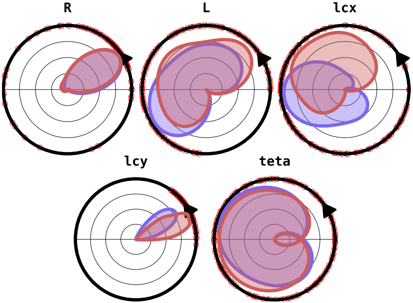


Figure – Polar distribution of data points in the parameter space for training. Data points from the power spectrum datasets are shown in blue, and those from the Gaussian spectrum datasets are shown in red.

Model

- $\frac{1}{R}$ as parameter to avoid the disruption of the standard normalization
- same model as the 1d model
- more parameters to predict δn_0 and τ_0

Results

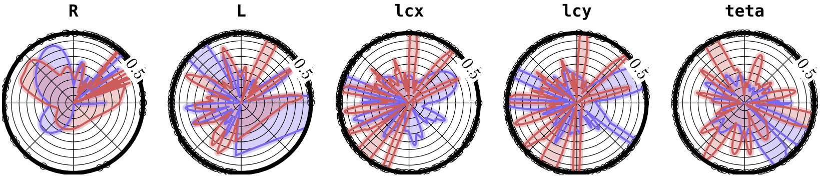


Figure – The mean residuals for every value of every parameter, in blue filled we got the residuals for the gaussian datasets, and in red the residuals for the power spectrum datasets.

- very good performances with a R^2 of 0.92 for the gaussian sets and 0.89 for the power spectrum sets
- better than all the previous models for this case
- no particular strange tendency in the residuals
- high θ is more difficult to tackle as intended

Physical Results

- the model is able to predict correctly τ_0 and δn_0
- is it able to tackle specific physical cases of this problem and how the prediction quality evolves in non-linear regime ?
- mean delay study, amplitude and standard deviation of the delay dependency

Mean Delay Study

- in the non-linear regime for the 1d analytical model the mean delay verified :

$$\langle \tau \rangle = \tau_0$$

- Due to the non one step like shape of the perturbation, this is not verified (FHTP theory)

$$S(x) \approx \frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{L - x - a(l_c x, L, \sigma)}{b(l_c x, L, \sigma)}\right)$$

- So $\langle \tau \rangle$ should decrease with the amplitude of the perturbation

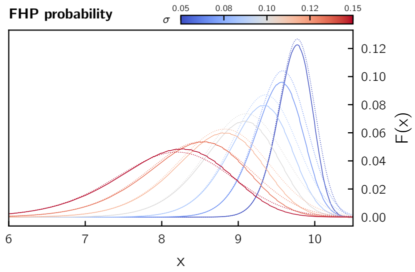


Figure – the numerical calculation of $F(x)$ the First Time Hitting probability (FHP) for numerous σ in plain line and the approximated formula in dotted line.

- $\frac{1}{R} = 4$ to see the difference of efficiency between the 1D and 2D models
- very good prediction of τ_0 with the 2d model centered and with a small standard deviation of the residuals.
- other models tends to locate the cut-off layer too close from the antenna

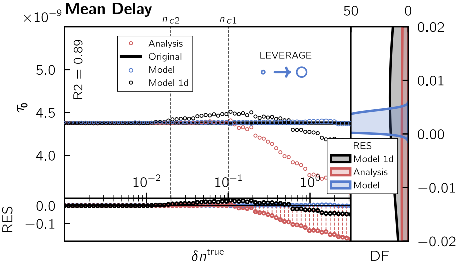


Figure – the linear and the 1d, 2d model prediction for τ_0 over the δn_{true} the analytic model prediction corresponds to the $\langle \tau \rangle$ value

Amplitude Prediction Study

- see the evolution of prediction quality of δn_0 with increasing of the amplitude
- the best model is the 2d model with a very qualitative prediction in non-linear regime (residues are centered and have a small standard deviation)
- step-like prediction seems to be due to the gridded training for small amplitude.
- other models are more erratic and collapse in non-linear regime

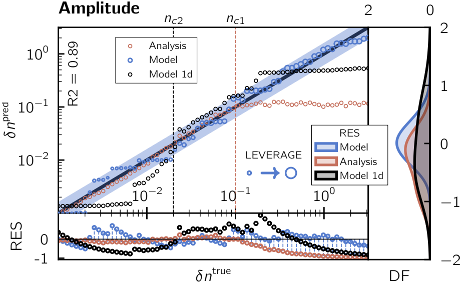


Figure – the linear and the 1d, 2d model predictions for δn_{pred} over the δn_{true} the analytic model prediction corresponds to the $\langle \tau \rangle$ value

Standard deviation of delay Study

- with the analytical model we observed a linear dependency of the standard deviation of the delay with the amplitude of the perturbation
- the characteristic saturation of the standard deviation in non-linear regime is well predicted by the 2d model

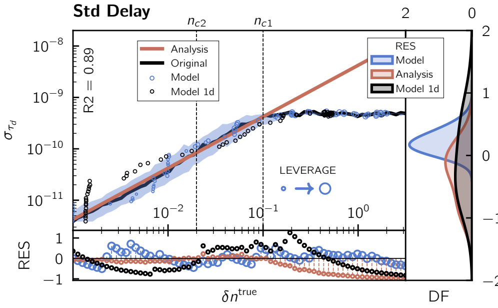


Figure – Plot of σ_{τ_d} over the linear and the 1d, 2d model predictions for δn_0. The simulation set-up is the same as the fig 15

Conclusion

- more general 2D model was proposed handling well the non-linear regime and its specificities.
- significantlly better prediction of the amplitude and delay in the non-linear regime.
- need some insights about the turbulence field to apply this model
- results are 1 point bellow without these characteristics hardly obtained with other reflectometry/scattering techniques.

- recent improvements have been done changing the normalization technique to fit experimental requirements.
- thanks to the SPC and the ENS for the opportunity to work on this project